Appendix of "Real-World Light Field Image Super-Resolution via Degradation Modulation"

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In this Appendix, we prove the *commutative law of convolution and sampling*. According to Eq. 3 in the main body of our paper, we set the sampling grid to $\frac{\epsilon}{\alpha}$. Then, we can obtain

$$[\mathcal{I}_{real}(h,w)]_{\frac{\epsilon}{\alpha}} = \int_{h-\frac{\epsilon}{2\alpha}}^{h+\frac{\epsilon}{2\alpha}} \int_{w-\frac{\epsilon}{2\alpha}}^{w+\frac{\epsilon}{2\alpha}} \mathcal{I}_{real}(x,y)dxdy. \tag{I}$$

Substitute Eq. 1 into Eq. I, we can obtain

$$[\mathcal{I}_{real}(h,w)]_{\frac{\epsilon}{\alpha}} = \int_{h-\frac{\epsilon}{2\alpha}}^{h+\frac{\epsilon}{2\alpha}} \int_{w-\frac{\epsilon}{2\alpha}}^{w+\frac{\epsilon}{2\alpha}} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} k(u,v) \cdot \mathcal{I}_{ideal}(x-u,y-v)dudvdxdy. \tag{II}$$

By performing *integration by parts* on Eq. II to exchange the integration order of $dudv$ and $dxdy$, we can obtain

$$\begin{aligned}
[\mathcal{I}_{real}(h,w)]_{\frac{\epsilon}{\alpha}} &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left[\int_{h-\frac{\epsilon}{2\alpha}}^{h+\frac{\epsilon}{2\alpha}} \int_{w-\frac{\epsilon}{2\alpha}}^{w+\frac{\epsilon}{2\alpha}} k(u,v) \cdot \mathcal{I}_{ideal}(x-u,y-v)dxdy \right] dudv \\
&= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} k(u,v) \cdot \left[\int_{h-\frac{\epsilon}{2\alpha}}^{h+\frac{\epsilon}{2\alpha}} \int_{w-\frac{\epsilon}{2\alpha}}^{w+\frac{\epsilon}{2\alpha}} \mathcal{I}_{ideal}(x-u,y-v)dxdy \right] dudv \\
&= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} k(u,v) \cdot [\mathcal{I}_{ideal}(h,w)]_{\frac{\epsilon}{\alpha}} dudv
\end{aligned} \tag{III}$$

According to the definition of convolution, Eq. III can be rewritten as

$$[\mathcal{I}_{ideal}(h,w) \otimes k(h,w)]_{\frac{\epsilon}{\alpha}} = [\mathcal{I}_{ideal}(h,w)]_{\frac{\epsilon}{\alpha}} \otimes k(h,w) \tag{IV}$$

That is, the order of convolution and sampling can be exchanged.